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REVIEWS

Gas Dynamics. By M. H. AKSEL and O. C. ERALP. Prentice Hall. 1994. 461 pp. ISBN 0 134 97728 9.
Compressible Fluid Dynamics – With Personal Computer Applications. By B. K. HODGE and K. KOENIG. Prentice Hall. 1995. 634 pp. ISBN 0 133 08552 X.

Both these books, which contain many worked examples based on modern engineering problems, present the material usually included in gasdynamic courses for undergraduate mechanical engineers, namely basic thermodynamics, one-dimensional isentropic flow and shock waves, Fanno, Rayleigh and isothermal flows, and use of oblique shocks and Prandtl–Meyer waves for the treatment of simple two-dimensional flows. *Gas Dynamics* by Aksel & Eralp presents the material in the conventional order listed above with the addition of an interesting introduction to detonation and deflagration waves in the section on Rayleigh flow.

Compressible Fluid Dynamics by Hodge & Koeing is aimed at both mechanical and aeronautical engineers and includes chapters on the equations for two- and three-dimensional flows, on their solution by the method of characteristics, and on high-temperature gas dynamics. The order and presentation of the material is different and refreshing. Thus the study of simple two-dimensional flows by oblique shocks and Prandtl–Meyer waves is introduced before the treatment of one-dimensional flow. This approach highlights the importance of waves in supersonic flows and should stimulate student interest. The initial treatment of one-dimensional flow is based on a completely generalized approach followed by the treatment of special cases (that is, area change, heat addition etc.). Furthermore the inclusion of a disc containing a program for the calculation of general one-dimensional flows allows students to appreciate the accuracy of the various assumptions made. Programs for the calculation of the compressible flow functions and for the solution of two-dimensional and axisymmetric flows by the method of characteristics are also included on the disc and used in the worked examples. The treatment of the method of characteristics is very clear but more discussion of the nature of elliptic and hyperbolic equations and the associated boundary conditions would be useful. The text is marred by a number of misprints but overall this is a ambitious book which deserves a place on the bookshelves near, but not replacing, *Elements of Gasdynamics* by Liepmann & Roshko (Wiley, 1957).

L. C. SQUIRE

Applications of Padé Approximation Theory in Fluid Dynamics. By A. POZZI. World Scientific, 1993. 286 pp. ISBN 9 810 214146 1. £28.

Interest in the application of Padé approximants in many areas of mathematical physics has intensified over recent years. This has been especially true in fluid mechanics, and the appearance of a book describing modern developments and applications is therefore particularly timely. The basic idea of Padé approximants is that a single Taylor series expansion can be recast in the form of a quotient of polynomials in order to provide a better representation of the complex singularities of the function being approximated, thereby extending the range of validity of the approximation well beyond the radius of convergence of the original power series. The attractiveness of this approach in fluid mechanics is obvious, since it provides the possibility of extending some early-time or short-range asymptotic series so as to describe a much greater range of values, possibly even right out to infinity.

In part 1 of the book Pozzi gives a clear introduction to the basic concept of Padé approximants, and provides some convincing examples of their effectiveness for functional approximation (e.g. $\log(1+x)$ can be approximated accurately for $0 < x < 10$ by a Padé approximant involving just a quotient of cubic polynomials). Part 2 is devoted to a derivation of the Navier–Stokes equations, which is rather too brief and sketchy to be of use to the reader, but which has presumably been included here in the interests of some sort of completeness. However, it is in the final two sections that the author gets down to describing the use of Padé approximants in steady fluid mechanics (part 3, which forms the majority of the book) and in unsteady fluid mechanics (part 4). The main consideration in part 3 is the convective flow close to heated bodies, and quite a wide range of examples drawn on the whole from the author's own research are given. One particularly impressive application concerns the convective flow over a thin plate with a constant temperature maintained on one side; the Taylor expansion in small $x^{1/2}$ (x the longitudinal coordinate) soon diverges downstream, but when rewritten as a Padé approximant yields an approximation which is valid for all x. Another interesting application involves determination of solutions for the Bickley jet in a *moving* external stream.

In the rather too brief part 4 the application of Padé approximants to the problem of determining the boundary layer flow on an impulsively started cylinder (as studied by Cowley 1983) is described. Most importantly, Cowley demonstrated that the Padé approximants predict a finite-time singularity in the boundary-layer equations, in agreement with earlier hypotheses, although unfortunately this point is not mentioned.

As has already been indicated, this book has a number of serious shortcomings, particularly in the rather sketchy way in which the author describes some of the fluid mechanics, together with being marred by a number of typographical errors (e.g. 'odograph' instead of hodograph in the chapter title on page 107). Having said this, however, the author is to be congratulated on communicating at least a flavour of the very rich variety of applications which have been made, and of the possibilities for further developments. The book certainly repays careful study, and is therefore to be recommended.

REFERENCE

Cowley, S. J. 1983 Computer extension and analytic continuation of Blasius' expansion for impulsive flow past a circular cylinder. *J. Fluid Mech.* **135**, 389–405.

N. Peake

The Mathematical Theory of Dilute Gases. By C. Cercignani, R. Illner and M. Pulvirenti. Springer, 1994. 347 pp. ISBN 3 540 94294 7. DM 84.

The kinetic theory of dilute gases,† as formulated by Maxwell and Boltzmann and subsequently used by Chapman and Enskog to predict accurate values of viscosities, conductivities and diffusivities for such media, can rightly be described as a successful theory. One should also recall its prediction of the existence of thermal diffusion before this phenomenon had been observed in nature. In recent times the theory attracted substantial interest from people working in the field of very-high-speed, very-high-altitude flight in (or in the fringes of) the atmosphere, in the behaviour of plasma, and in the various applied sciences that deal with mixtures of gases.

† Broadly speaking, those for which the average distance travelled by a molecule between successive collisions with other molecules is very much larger than the molecular diameter.

The kinetic theory that is the subject of the present book is built around the nonlinear integro-differential equation, usually called the Boltzmann equation in honour of his prime role in its development, that governs the behaviour of the one-particle molecular-velocity distribution function f. It is not too difficult to give plausible derivations of the Boltzmann equation, and then to argue empirically for its validity by appealing to the quite remarkably accurate predictions that can be made with it. To give a rigorous account of the logical status of the Boltzmann equation is quite another matter, and a very proper topic for investigation by modern mathematical analysts.

Written by Professor Cercignani (who has already written extensively on the mathematics of kinetic theory) and two colleagues, the book contains a coherent account of advances in mathematical kinetic theory during the past twenty years or so. It discusses and explains a number of topics but puts some special emphasis on the matter referred to in the previous paragraph. Chapter 1 begins by describing what is meant by a gas, and goes on to give a brief but interesting history of kinetic theory. Chapter 2, entitled 'Informal introduction to the Boltzmann equation', introduces the reader to the $6N$-dimensional phase space that must be used to account for spatial positions and velocities in an array of N identical hard spheres (the majority of the book is confined to the treatment of this elementary but, with care, nonetheless useful molecular model), as well as to the Liouville equation that governs the character of a probability-density function $P(z,t)$ for points in the $6N$-dimensional space. The appearance of probability functions like P and f makes the connection between dynamics and thermodynamics that has such far-reaching consequences for the treatment of systems of large numbers of particles (N is typically of order 10^{19}, or 10^{23}, depending on whether one likes to imagine numbers of molecules in cubic centimetres of real space, or in moles of material). The probability that the system is in some chosen domain of phase-space is given by the integral of P over the domain, and incorporates "...the assumption that the measure defining the probability is absolutely continuous with respect to the Lebesgue measure". Evidently the 'informality' of the chapter's title is accompanied by some careful mathematical groundwork on which to build subsequent discussions, and the reader is given a signal of what is to come.

The Liouville equation, which is concerned with information about every particle, forms the basis from which a rigorous derivation of the Boltzmann equation for the single-particle function f can be undertaken. It is during the coarsening of the level of detail in the description of a system from the Liouville function P towards the Boltzmann function f that one meets the need to describe encounters or collisions between molecules and begins to see the necessity for the assumption originally, and picturesquely, described by Boltzmann as the assumption of 'molecular chaos'. The book spends some time, in Chapter 3, on what are called, in the text, "...the main properties of the solutions of the Boltzmann equation...." and, in the chapter's title, "Elementary properties of the solutions". Apart from developing the level of mathematical rigour that is one of the book's main themes, this chapter also introduces, and fruitfully discusses, a number of physically significant items, including a few of the famous 'paradoxes' that have accompanied Boltzmann's ideas down the years. Amongst the most useful items from the point of view of the applied scientist one can quote the derivation, from a basis of Boltzmann's equation, of the equations of change, as some people call them, or the conservation equations, as they are described by others, or the macroscopic balance equations, which is the name preferred by the present authors, and one should not overlook Boltzmann's so-called H-theorem that has vital things to say about the nature of entropy, its production and transfer, in non-equilibrium systems that are not far removed from states of local equilibrium.

Of course this early material is not new, although it is good to have this fresh view of it, and of its emerging connections with mathematical exactitude. The latter is the strong strand that runs through the remainder of the book. Chapter 4 is devoted to a close examination of the validity of the Boltzmann equation; amongst other things it confirms that the irreversible Boltzmann equation can be rigorously derived from reversible mechanics. Chapter 5 discusses questions of existence and uniqueness, while Chapter 6 examines the initial value problem for the homogenous Boltzmann equation. Chapter 7 looks for solutions for the function f that are close to the classical, equilibrium, Maxwellian form; under these circumstances it is legitimate to linearize the collision operator, and so much of the chapter is devoted to studies of its spectral properties. The book concludes with four short chapters on boundary conditions, existence results for initial-boundary and boundary value problems, particle simulation of the Boltzmann equation, and a discussion of hydrodynamic limits which makes enlightening use of the sort of coordinate scalings that are familiar from asymptotics, or boundary-layer theories. The book concludes with five pages on open problems and new directions, in which the authors describe the topics that their considerable collective experience tells them are worthy of close attention.

The book is by no means an easy read for those not versed in the language and methods of up-to-date analysis. In view of the density of the mathematical arguments a case could be made for the inclusion, in future editions of the book, of an index of 'pages of first appearance and definition' of the many symbols that must be used in such a text. Notwithstanding these comments this is a volume that will repay close study by those who wish to catch up with the present state of the art in the mathematical theory of dilute gases.

J. F. Clarke

SHORT NOTICES

An Exploration of Chaos. By J. Argyris, G. Faust and M. Haase. North Holland, 1994. 751 pp. ISBN 0 444 82002 7 (hardback) Dfl.445.00, US$254.25 and ISBN 0 444 82003 5 (paperback) Dfl.175.00, US$100.00.

This book introduces chaos with a detailed description of many of the most central topics in this subject area. A fairly high level of mathematical capability together with some familiarity with theoretical physics appears to be needed fully to appreciate the text, yet so many examples are introduced from a relatively elementary level that a wider readership can benefit. Dynamical systems, with and without dissipation, bifurcation theory and routes to chaos are prominent. So too, are references to turbulence and fluid experiments as well as other applications, though these are sometimes introduced by analogy rather than by complete correspondence with the concepts being explained. In addition to the detailed descriptions, there is very full referencing, with over 600 references, and the book is particularly distinguished by the large number of specially prepared high quality illustrations which greatly assist the reader. The enthusiasm of the authors and their associates for the subject is apparent.

Discharge Characteristics. Edited by D. S. MILLER. Balkema, 1994. 249 pp. ISBN 90 5410 180 6. £72.00.

The essence of this monograph is to provide designers of civil engineering hydraulic systems with data and theory from which to estimate the flow discharge and head losses at obstacles in the flow. There are three major chapters. One on discharge measuring structures concerns free surface flows over various weirs or through constrictions in a channel. The largest chapter, almost half the book, covers a wide range of obstacles in channels including weirs, sluice gates, bridge piers, expansions and contractions, trash racks, etc. The next substantial chapter is on flow in closed conduits with considerable emphasis on the effects of bends, combination of bends, and branches as well as more obvious structures such as valves. The remaining short chapters concern river diversion in closed conduits and flow through and over 'rockfill structures'. As may be expected, the book has a multitude of formulas and many graphs. There are five pages of references.

Energy Dissipators. Edited by D. L. VISCHER and W. H. HAGER. Balkema, 1995. 201 pp. ISBN 90 5410 198 9. £60.00, $85.00, or Hfl.150.

The topic here is biased towards large-scale energy dissipation such as is required for the flow being discharged below dams, although most of the data necessarily come from smaller-scale flows. The essence of flow dissipation is the promotion of strong turbulence and/or the breakup of water flows into drops which are small enough to suffer significant air drag. Much of the book is descriptive of various flow devices, as indicated by the chapter headings: Hydraulic jump, Stilling basins, Drop structures and plunge pools, Trajectory basins, Cascades, Drops and rough channels and Vortex drops. Parts of the book include much data on flows and discharge properties, e.g. in the discussions on hydraulic jumps and stilling basins; other parts are more descriptive and relate to difficulties in design at the largest scales. This applies particularly to erosion of unlined plunge pools. There are ten pages of references.

Thermofluids – An Integrated Approach to Thermodynamics and Fluid Mechanics Principles. By C. MARQUAND and D. CROFT. Wiley, 1994. 403 pp. ISBN 0 471 94184 0. £19.95.

This integrated approach to thermal systems provides a good introduction to elementary thermodynamics and fluid mechanics. There are a large number of worked examples including a thorough treatment of the steady flow momentum and energy equations, although in most cases the fluid is treated as incompressible.